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SECONDARY LOSS OF STABILITY OF AN EULER ROD

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The solution of the classical problem of the postcritical behavior of a compressed, simply supported rod is considered. The stability of the well-known elastic solution that emanates from the first critical point is analyzed and new branches of the states of equilibrium are found. The value of the force that corresponds to the secondary loss of stability is determined.

Until recently, the existing Lagrange solution of the problem of the postcritical behavior of an Euler rod in elliptic integrals has been considered to be complete [1–3] and has been used as the standard to verify numerical methods of solving nonlinear problems for thin-walled structures. Using a finite-element shell model that takes into account large displacements and rotations, Korobeinikov [4] found that if the symmetry condition is ignored, the secondary bifurcation of the solution occurs in the region of postcritical deformation, and an asymmetric deformation branch appears in the range of loads $2P_{\rm cr} < P < 7P_{\rm cr}$ ($P_{\rm cr}$ is the critical Euler force); this branch disappears with further increase in the axial force.

Figures 1 and 2 show the solution of the problem that was obtained by a numerical algorithm of determining the multiple-valued nonlinear solutions of rod flexure problems in the presence of many bifurcation and limiting points [5]. Here W and U are the mid-span deflection and longitudinal displacement of one end of the rod (the other end is immovable), respectively, and L is the rod length. Solid curves refer to stable states, and dashed curves to unstable states. In essence, two projections of the deformation curves in the $(N+1)$-dimensional space [5] onto the planes (P, W) (Fig. 1) and (P, U) (Fig. 2) are given for positive and negative values of the force. As numerical calculations have shown, the division of the rod into 28 finite elements provides high accuracy of nonlinear solutions for a wide range of rod flexure.

Using the Sylvester criterion, we established that the Lagrange solution [sections of the curve AB_1 (AB_2) and its smooth continuation in Figs. 1 and 2] is stable only up to the point B_1 (B_2), where $P = 2.18P_{\rm cr}$. A new branch of the unstable states of equilibrium B_1B_3 (B_2B_4) emanates from the point B_1 (B_2), where the two hinges are brought into coincidence and the rod can rotate as a rigid body. It is noteworthy that the branches B_1B_3 and B_2B_4 are the closed curves in the $(N+1)$-dimensional space which do not intersect with one another. According to the Lagrange solution, the force value at which the ends of the inextensible rod coincide is determined from the relations

$$2E(\pi/2,k) - F(\pi/2,k) = 0, \qquad P = (4/\pi^2)F^2(\pi/2,k)P_{\rm cr}.$$

Hence, $k = 0.90891$ and $P = 2.18338P_{\rm cr}$. Here $F(\pi/2,k)$ and $E(\pi/2,k)$ are complete elliptic integrals of the first and second kinds and k is the additional modulus of the elliptic integral [6]. The solutions B_1B_3 (B_2B_4) exist for $-2.18P_{\rm cr} < P < 2.18P_{\rm cr}$. For $2P_{\rm cr} < P < 10P_{\rm cr}$, any other equilibrium curves branching from the Lagrange solution were not found. Thus, as a series of stable states of equilibrium for increasing load, the curves $0A$ and AB_1 (AB_2) are realized with a subsequent jump onto the branch CD where the rod is straight again, but the movable support is at the other side relative to the immovable support compared to the initial position of the rod.

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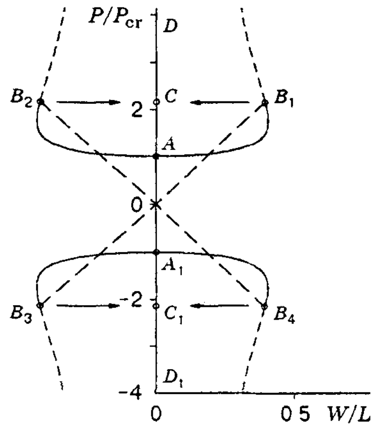

Fig. 1

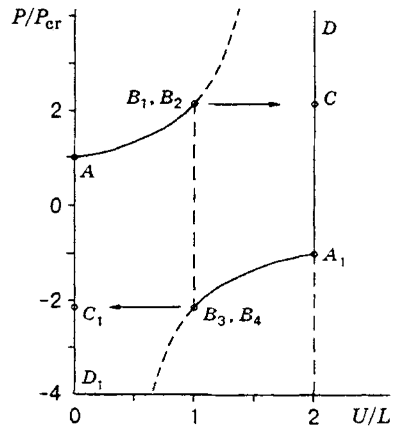

Fig. 2

The results of the qualitative experiments on a celluloid model have shown that the states of equilibrium that correspond to the branch AB_1 (AB_2) after the point B_1 (B_2) (loop-like shapes) cannot be realized without additional restrictions on the displacements of the rod. This fact supports their instability.

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